In the Head Calculations - Introduction

Skills for the adult world

Adult students come to numeracy class expecting to relearn all of the formal, pen and paper methods of calculation from their school days. However, in these days of calculators, spread sheets and computerised cash registers, it is just as important to be able to use 'in the head' or informal methods, and to be able to estimate and use a calculator efficiently. For these informal methods, what is written on paper will not a neat and orderly 'sum', but a few 'jottings' to help the shortcut or 'back-of-envelope' calculations.

These informal or 'in-the-head' approaches are powerful and important in the adult world. In industry and workplace settings estimation and quick checks of calculator and spread sheet results are essential to ensure that you don't get nonsense from the technology. Mistakes leading to wasted materials and labour are expensive in a work situation.

Informal approaches are not only important in the workplace. Many adults use them without thinking for rapid figuring of their capacity to save money from their annual salary or to pay off loans and bills.

Creative and individual strategies

Others adults, with less confidence in numeracy, have developed individual strategies, of which they are quite proud, to get around using the operations that they did not feel confident with at school. For example, there are many adults who tend to avoid multiplication, preferring instead to use repeated addition; these people get very fast and skilled at addition. There are many, many more who would never even think of trying division: somehow multiplication or repeated addition will get them close enough to the answer they need.

Boosting student confidence

Introducing shortcuts and practice at using 'in the head' techniques in the numeracy class can make adults feel really empowered. It validates the strategies they have developed for themselves which previously they would not think legitimate in a maths classroom. They can share their own methods and display their creativity, as well as expanding their repertoire of techniques. Once you help students acquire new strategies it is amazing how they will suddenly find uses for them, or bring to class stories of sharing them with others.

Some of the strategies in this section tap into students' existing skills, such as rapid addition, and allow them to extend their repertoire to calculating percentages without having to use the algorithms that they did not feel comfortable with. At the same time, it should be kept in mind that students will benefit from learning the multiplication facts if they can. Some tips for helping students do this are included in the *Exploring Numbers* section of this resource.

Numbers sense not rote procedures

In the head methods focus on ‘number sense’ rather than algorithms, rules and formulae. They build on students’ existing knowledge of number, encourage them to be more flexible in their calculations and eventually increase their understanding of how numbers work; their ‘number sense’. This in turn vastly increases their ability to function with numbers in the world and their confidence with numeracy.

This section provides some of the more common strategies for in the head, or shortcut calculation and some opportunities for practice. It is just a beginning and by no means exhaustive; hopefully other strategies can be shared and discussed with students as they arise incidentally in class.

Adapting strategies to other uses

Most of the strategies can be extended from their original use to be applied powerfully and easily in other common situations which may be relevant to students in or out of numeracy classes. For example, ‘counting on’ when represented as a sketch in the *Calculating Change* activity can be used for any subtraction, but is also particularly powerful for time calculations: working out ‘How long since…?’ or How long ago was…? or ‘How many hours of work should I be paid for?’

The strategy of splitting numbers for halving can also be used for heaps of percentage calculations, including 50%, 25%, 75% and even 2 ½ % and 17 ½ %, freeing learners from previously confusing, meaningless, percentage formulae (see the *Percentages* section).

Useful Number Pairs

Overview

This activity presents a number of strategies which will help students recognise and recall useful number pairs fundamental to 'in the head' addition and subtraction. These include pairs which total 10, such as, 2 & 8, 3 & 7 and pairs which total 100, such as, 20 & 80, 30 & 70, 75 & 25 or 35 & 65.

The activity contains strategies for exploring and learning the pairs initially and practising instant recall.

These number pairs are applied simple change calculations in the Activity *Calculating Change*. The two activities could be combined section by section if this way of thinking is new for students.

Skills and Knowledge

Recall of number pairs totalling 10 & 100

Preparation and Materials

- At least 10 counters
- At least ten 10 cent coins
- At least two 5 cent coins
- A 10 or 6 sided dice

Photocopy Practice Sheets 1 & 2 (1 per student)

Suggested Procedure

A reminder of number pairs that total 10

Several strategies can be used to revise the pairs of number which total 10:

Counters in the hand?

- Begin with a collection of 10 counters on the table.
- Take several in your hand.
- Close your hand.
- Ask students to count how many are left on the table and tell you how many must be in your hand.

Continue with different numbers of counters until you are sure that students confidently remember the number pairs.

If students do not already know these number pairs, give them their own a pile of 10 counters, ask them to make two separate piles and record the numbers of counters in each. Ask them to see how many different pairs they can make with the 10 counters.

Throw of a dice

- Throw a dice (preferably 10 sided, but 6 will do).
- Call the number you have thrown.
- Ask students to tell you the number needed to add up to 10.

The 10 sided dice also includes 0 which will draw attention to the 10, 0 pair.

It could be useful to practise this in pairs. Just give one dice to each pair of students and ask them to take turns throwing and calling the pairs.

Introducing Number grids - pairs totalling 10

Practice Sheet 1 contains several grids designed for students to develop instant recall of the number pairs totalling 10.

Distribute one copy to each student.

Explain:

- *These grids have lots of pairs of numbers that add to 10*
- *They also have one spare number*
- *Your task is find all the pairs and cross them off until you find the spare number*
- *The best way is to find each pair and draw a line connecting the two numbers*
- *Then put a cross or a circle round the numbers*
- *Keep doing this till you have just one number left in the grid*

Correcting solutions

There are many ways of matching the number pairs but if all the pairs are correct then only one number remains unmatched at the end.

Explain:

- *Check with another student to see if you have the same single number left over*
- *If you don't agree then both of you check back and make sure all of your pairs are correct*
- *It is useful for checking if you drew the lines between the pairs of numbers as you found them*

Ask students to complete several of the grids.

If anyone finishes the first three grids whilst others are still going, ask them to try and use the blank to create their own version. Others could perhaps try this at another time, or for homework.

Ask students to then get others in the class to test their grid by finding the one spare number as before. [If the grid has not been done correctly then many unmatched numbers will remain.]

Extension grids – pairs which add to the next ten

The last two grids on *Practice Sheet 1* extend the idea of pairs which add to 10 by recognising pairs which will add up to the next ten, such as 15 & 5 which total 20, or 24 & 6 which total 30.

Extending to pairs that total 100

Display a pile of ten 10 cent coins and explain:

- *I have ten 10 cent coins here in the 'kitty'*

Ask:

- *How much money is this?*
- *How many cents altogether?*

Explain:

- *I am going to use these coins to make pairs that total 100*

Count out 6 of the 10cent coins: *10c, 20c, 30c, … 60c*

Ask:

- *Now I have 60 cents over here*
- *How much will I have left in kitty?*

Record the pair 60 + 40 = 100 on the board.
Repeat this for a few more pairs.

Ask:

- *Write down all of the possible pairs that we could make with these 10 cent coins*
- *Knowing these pairs helps to work out change easily*
- *We will try some of these later*

The first two grids on *Practice Sheet 2* use these pairs only, and would be useful at this point if students need to consolidate these before progressing to the next stage.

Where to next?

At this point there are **two possible choices** depending on how challenging the students are finding the content so far.

If they are challenged they may need to spend more time consolidating these pairs and also the motivation of seeing how they apply to calculating change, before going on to further pairs. See *Calculating Change Activity* following.

If they are not yet challenged then extend the number pairs to those ending in 5 such as, 65 & 35, immediately.

Extending to pairs ending in 5 (eg 65 & 35)

Explain:

- *I am now going to change one of these 10 cent coins for two 5 cent coins*
- *How much money is in kitty now?*

[There is still $1 or 100cents in kitty]

Count out a pile of coins that ends with a 5 cent coin, such as:

- *10c, 20c, 30c, 35c*

Ask:
- *How much is left in the kitty now?*

Count the money out loud together to check students' answers.

Demonstrate and explain:
- *One way to work this out is to imagine the other 5 cents in the kitty*
- *35 cents here, and another 5 from kitty would be 40 cents*
- *We know that the pair for 40c is 60c*
- *So in kitty we have 60c and the other 5c*
- *That's 60c and 5c which is 65c*

Use the coins to go through this thinking process until students see it clearly.

Record the pair 35+ 65 = 100 on the board

Try a few more examples together. For example, ask:
- *If I take 55c from kitty how much is left?*
- *What about 25c?*

Keep counting these out with the coins until students are really clear about the thinking process.

Continue until all possible pairs are recorded.

When the pairs are all recorded on the board ask:
- *Can you see a pattern here that will help remember these pairs?*

Students may readily see a pattern, such as: *the tens part of these numbers always adds up to 9 (or 90) and the two 5s make up the extra 10 to get to 100*. Or they may need extra prompting to see it.

Exploring the pattern is worthwhile, because seeing the pattern makes remembering the pairs much easier than if they were rote learned.

Practice Sheet 2 has several examples of grids for consolidating these pairs.

Useful Pairs

Practice Sheet 1

In each of these grids
Cross out the pairs that total 10.
What number is left in the grid?

3	2	4	6	1
10	4	7	5	0
7	4	8	9	2
3	5	6	8	6
8	9	2	7	1

5	5	8	4	2
3	8	4	5	1
4	2	2	7	6
3	5	7	3	9
8	6	2	8	6

5	6	2	1	8
10	5	4	3	1
8	9	7	6	5
5	4	4	9	2
7	3	8	2	0

Use the blank to create a grid or your own

In this grid
Cross out the pairs that total 20

15	6	2	11	8
10	5	14	13	1
18	9	17	6	15
5	14	4	19	12
7	3	18	2	10

In this grid
Cross out the pairs that total 30

24	22	3	4	29
9	7	21	20	23
1	26	8	6	28
27	9	27	28	5
10	2	25	3	2

Useful Pairs

Practice Sheet 2

In each of these grids:

Cross out the pairs that total 100.
What number is left in the grid?
Use the blank to create your own

50	60	20	40	70
30	60	70	90	80
10	90	40	30	10
40	10	10	80	60
90	20	70	90	50

60	50	30	10	90
70	40	30	90	80
10	90	40	70	10
60	10	10	80	60
90	20	20	90	50

25	95	80	95	35
15	65	75	35	30
45	65	15	85	5
70	10	65	90	35
5	55	40	20	60

15	95	20	70	60
55	40	70	50	20
85	30	5	85	25
50	65	80	35	45
80	15	85	75	30

20	25	55	40	95
15	35	10	75	70
20	25	20	80	65
30	35	90	85	80
5	45	80	60	75

Calculating Change

Overview

This activity demonstrates how to use the number pairs which total 10 and 100 to perform simple, in the head change calculations.

It could be used in conjunction with *Useful Number Pairs* or as a separate follow up activity, depending on students' prior skills and knowledge.

It also describes the more general 'counting on' method for calculating change, which can be applied to a variety of situations that would otherwise use subtraction.

Skills and Knowledge

- Recall of number pairs totalling 10 & 100
- Calculation of change using number pairs
- Calculation of change by counting on

Preparation and Materials

Photocopy Practice Sheets 1 & 2 (1 per student) then cut as indicated into sets of 10 questions.

- A 10 or 6 sided dice
- At least ten 10 cent coins
- At least two 5 cent coins
- A collection of supermarket catalogues or advertisements

Suggested Procedure

Reminder of number pairs totalling 10

First revise with students the number pairs which total 10 from the last activity, but this time in the context of money, as pairs which combine to give 10 cents.

- Throw a dice (preferably 10 sided, but 6 will do).
- Call the number you have thrown as a number of cents eg 6 ce nts.

Ask

- *How many more cents would I need to add up to 10?*

Continue until students are confidently answering and all the number pairs have been revised.

Reminder of number pairs totalling 100

Write on the board:

30 + ? = 100
90 + ? = 100

Ask:

- *Write down all the other number pairs that add to 100*

List all of the pairs on the board in preparation for the next step.

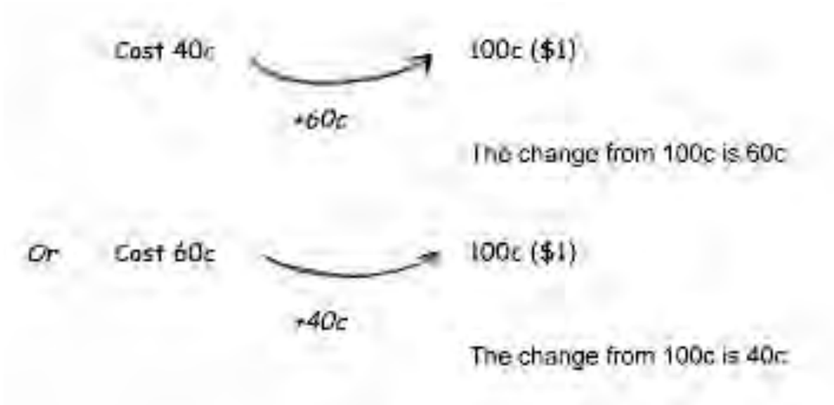
Calculating change from a dollar

Point to one of the number pairs, such as 60 + 40 = 100

Ask:

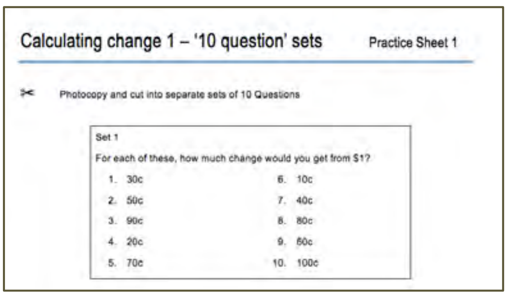
- *If I buy something worth 40 cents, how much change would I get from a dollar?*
- *If I pay 60 cents for something how much change would I get?*

Representing the situation in diagram form can be more powerful than words.



Repeat for a few of the other number pairs.

Practice Sheet 1 contains several sets of '10 questions' relating to change calculations using these number pairs. See description of Activity: *10 questions,* for suggestions on how to use 10 question sets with your group.



Calculating change 1 – '10 question' sets Practice Sheet 1

Photocopy and cut into separate sets of 10 Questions

Set 1

For each of these, how much change would you get from $1?

1. 30c	6. 10c
2. 50c	7. 40c
3. 90c	8. 80c
4. 20c	9. 60c
5. 70c	10. 100c

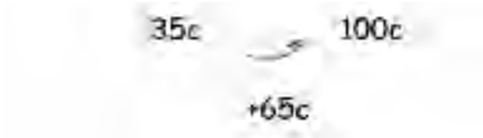
Extending to change from more than $1

Ask:

- *I buy something for 35 cents*
- *I pay with $1.*
- *How much change do I get?*

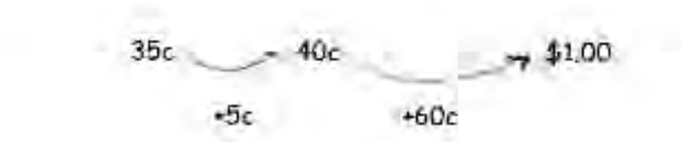
Remind students of the pair 35c & 65c that add to 100 cents or $1. The other part of the pair is the change you get, or how much more would take it to 100 cents.

If students have remembered the number pairs ending in 5 the diagram would be:



The change from 100c is 65c

If students **do not** have recall of the number pairs ending in 5 then the diagram can be broken into further steps:



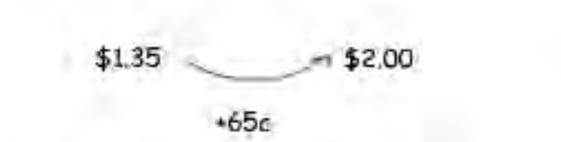
The change is   5c + 60c = 65c

Ask:

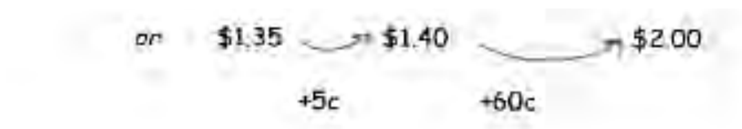
- *What if I buy something worth $1.35*
- *I pay with $2*
- *How much change now?*

Assist students to see that it is almost the same situation.
We have paid the one whole dollar already then another 35 cents. The change will be how much more we need to get to the next dollar? The gap between the amounts is the same as before.

In diagram form:



The change from 100c is 65c



The change is   5c + 60c = 65c

Try more with the 35 cents until you are sure that students have the idea of using the pair that takes you to the next dollar.

For example, ask:

- *What if I buy something worth $4.35*
- *I pay with $5*
- *How much change now?*

- *What about something worth $9.35*
- *I pay with $10*
- *How much change now?*

These diagrams are visual representations of the 'counting on' method for giving change that was regularly used before computerised cash registers did the thinking for the person on the till. For some students, understanding of the thinking involved in this method would be enhanced by going through the physical actions of giving change this way. Setting up a shop or market stall role play can help.

Extending beyond the simple number pairs

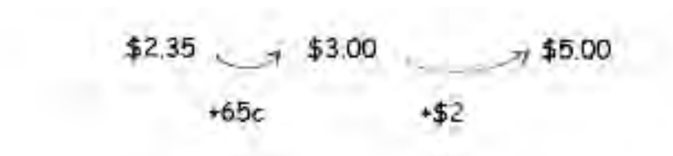
The next step involves amounts that involve extra dollars in the change.

For example:

- *What if I buy something worth $2.35*
- *I pay with $5*
- *How much change now?*

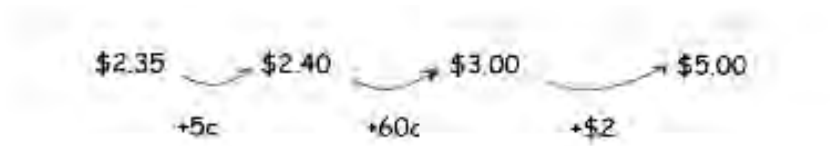
Encourage students to count on to the next dollar $2.35 cents and the pair 65 cents takes us to $3. Then we need $2 more to get to the $5.

If using recall of the pairs it would be represented visually by:



The change is 65c + $2 = $2.65

Or without recall of the pairs:



The change is 5c + 60c + $2 = $2.65

Try a few more together, for example:

- *What's the change from $5 if you spend:*
  $3.50, $2.45, $3.15, $1.85, $2.95?

Practice Sheets 2 contain sets of change calculation '10 Questions' which progress in levels of difficulty. See description of the *10 questions* Activity for suggestions on how to use these with your group.

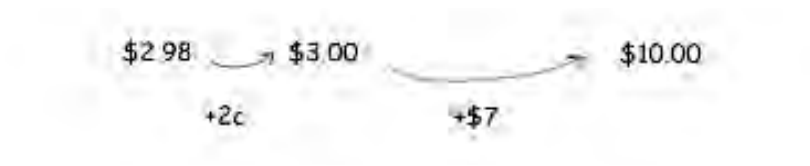
Extension - amounts which don't end in 5

Obviously the second method described above can be used to calculate any amount of change owing. However, it is complicated by the fact that only 5 cent coins are available to give as change. For numeracy in the real world this situation should be discussed with students.

Pose a question such as the following:

- *You buy an item worth $ 2.98 what change do you expect if you pay with $10?*

The diagram would be:



The change owing would be 2c + $7 = $7.02

Ask:

- *What would happen?*
- *How much change would you get?*

Discuss this with students and look at shopping catalogue advertisements or catalogues together.

Investigate how often these kinds of prices appear.

Ask students why they think prices such as $1.99 are popular.

Discuss what happens in reality in the supermarket with small numbers of cents.

Further Practice

Distribute catalogues or leaflets to pairs of students.

Explain:

- *Select one item that you might buy*
- *Decide what coin or note you would use to pay with*
- *The work out how much change you would get.*

When individual students can calculate the change for one item easily, move them on to two or three items at a time.

Other uses of counting on

The method of 'counting on' demonstrated in this activity can be applied to calculations involving time. See the *Calculating Time* activity.

It can also be applied as an alternative method of subtraction which avoids the type of mistakes commonly made by adults who have been confused by the formal school methods.

Calculating change 1 – '10 question' sets Practice Sheet 1

✂ Photocopy and cut into separate sets of 10 Questions

Set 1

For each of these, how much change would you get from $1?

1. 30c	6. 10c
2. 50c	7. 40c
3. 90c	8. 80c
4. 20c	9. 60c
5. 70c	10. 100c

Set 2

For each of these, how much change would you get from $1?

1. 60c	6. 20c
2. 30c	7. 60c
3. 50c	8. 100c
4. 80c	9. 10c
5. 70c	10. 90c

Set 3

For each of these, how much change would you get from $1?

1. 95c	6. 65c
2. 55c	7. 45c
3. 15c	8. 5c
4. 75c	9. 85c
5. 35c	10. 25c

Calculating change 1 – '10 question' sets Practice Sheet 2

✂ Photocopy and cut into separate sets of 10 Questions

Set 4

For each of these, how much change would you get from $2?

1.	$1.50	6.	$1.20
2.	$1.85	7.	$1.35
3.	$1.65	8.	85 cents
4.	90 cents	9.	$1.95
5.	$1.05	10.	55 cents

Set 5

For each of these, how much change would you get from $5?

1.	$3.50	6.	$1.35
2.	$4.75	7.	$2.90
3.	$1.50	8.	$4.10
4.	$2.25	9.	$2.50
5.	$4.40	10.	$1.10

Set 6

For each of these, how much change would you get from $10?

1.	$4.50	6.	$8.30
2.	$9.25	7.	$2.50
3.	$7.50	8.	$4.90
4.	$5.70	9.	$7.25
5.	$4.60	10.	$6.35

Calculating change 1 – ‘10 question’ sets Practice Sheet 2 (cont.)

✂ Photocopy and cut into separate sets of Mixed10 Questions

Mixed Set 1

1. What is the change from $1 if you spend 35c?
2. You have $3.15. How much more do you need to make $4?
3. $1.55 + 45c = …………
4. What is the change from $5 if you spend $3.40?
5. You want $20 but only have $19.75. How much more do you need?
6. $100 is needed but you only have $65. How much more do you need?
7. 45 c and 55c equals …………
8. You spend $1.25 then 75 cents. How much do you spend altogether?
9. You buy a cake for $2.35 and pay with $5. How much change will you get?
10. The total of $3.65 and $1.35 is …………

Mixed Set 2

11. What is the change from $2 if you spend $1.45c?
12. You have $2.15. How much more do you need to make $3?
13. $1.65 + 35c = …………
14. What is the change from $10 if you spend $8.30?
15. You want $10 but only have $9.25. How much more do you need?
16. $100 is needed but you only have $65. How much more do you need?
17. 85c and 15c equals …………
18. You spend $4.75 then 25 cents. How much do you spend altogether?
19. You buy a cake for $1.95 and pay with $5. How much change ?
20. The total of $2.35 and $2.65 is …………

Subtraction using Counting On

Overview

This activity extends the 'Counting On' method of calculating change using 'Useful Number Pairs' to apply to any subtraction, including those involving time.

Ideally it should follow the activity *Calculating Change.*

Skills and Knowledge

Subtracting using counting on
Subtracting using a time line (years)
Subtracting using a time line (hours)

Preparation and Materials

Photocopy Practice Sheets 1, 2 & 3
(1 per student)

Suggested Procedure

Introducing the skill

Explain that you are going to have another look at the method that the class learned to calculate change and see how it can be used for subtractions that don't involve money.

Set the skill in context with the following scenario:

A group of friends decide to ride 100km to raise money for charity.
After 2 hours Alisha had ridden 28 km and she was tired.
How much further did she have to ride?

Write on the board:

28 *100*

Ask the following scaffolding questions and fill in the steps on the diagram as you go:

- *What's the first step we could use to count on to 100?*
- *What's the first useful number?*
- *How many km to get to 30?*
- *What's the next step we could take?*
- *How many km to get there?*
- *So what's the total distance she has to ride?*

$$28 \xrightarrow{+2} 30 \xrightarrow{+70} 100$$

The distance left is 2km + 70km = 72km

Example 2: *Georgia had ridden 36km when she got tired. How far did she have left to go?*

Ask:

- *How do we start the diagram this time?*
- *What number goes first?*
- *What's the next step?*
- *How many kms to get there?*
- *And the next step?*

$$36 \xrightarrow{+4} 40 \xrightarrow{+60} 100$$

Total left to ride: 4km + 60km = 64 km

Ask students to try a couple more examples on their own before you check them on the board with the group:

- *Josh had ridden 43 km when he got a flat tyre. How far does he have left to ride?*
- *Leanne stopped for a rest after 51 km. How much further does she have to go?*

Emphasise that this is subtraction

It is probably a good idea to remind students that what they have been doing is the same as subtraction. That way they will be more able to transfer the method to other situations.

Ask:

- *Do you know how these calculations would be written if they were school 'sums'?*

Explain:

- *We have been finding is how far* ***left*** *to go*
- *That's the same as the* ***difference*** *between two distances*
- *Yes it is* ***subtraction****.*

On the board:				
	100	*100*	*100*	*100*
	<u>- 28</u>	*<u>- 36</u>*	*<u>- 43</u>*	*<u>- 51</u>*

Ask:

- *Would these calculations be hard or easy as school subtraction sums?*
- *Which method do you find quicker - easier?*

Discuss the difficulties involved in carrying or decomposing numbers when using numbers like 100, 200 etc and compare to the method they have been using.

Checking the answers

Explain:

- *We have just done four subtraction calculations*
- *Usually you check the answers for subtractions*

- *What's the easiest way to do that?*
- *Yes its by doing the opposite – by adding*

On the board:

100 – 28 = 72   Check: 72 + 28 = 100 → 72 is correct

100 – 36 = 64   Check: 64 + 36 = 100 → 64 is correct

Ask students to check the other two calculations themselves.

Extending beyond one hundred

To extend the method beyond a single hundred let students try a few short calculations which are designed to 'scaffold' them through the steps as the numbers get bigger**.**

Examples:

100 – 70	*100 - 58*
200 – 70	*200 - 58*
600 – 70	*700 - 58*
1,000 – 70	*1,000 - 58*
2,000 – 70	*3,000 - 58*

When they have had a chance to try them out, go over the steps and the process of checking to ensure their procedures make sense.

Then get students to try a few more examples without the scaffolded steps.

For example:

500 – 87   600 – 45   800 – 83

Further examples of this type are available in *Practice Sheet 1.*

Extending to more complicated numbers

Pose the following question to the group:

- *A family has to travel 355 km to visit their relatives in another city. They stop for lunch after 146km. How much further do they have to drive?*

Ask students to tell you the steps as you build the diagram together:

4   50   100   50   5

146   150   200   300   350   355

Total left to travel: 4 + 50 + 100 + 50 + 5 = 209 km

Try a few more distance scenarios until students are confident that they could do any subtraction like this.

For example:

- *The total distance is 623Km they have gone 419km. How far left to go?*
- *The total distance is 904Km they have gone 589km. How far left to go?*
- *The total distance is 1,260Km they have gone 967km. How far left to go?*

Further distance and other subtraction calculations are provided in *Practice Sheet 2*.

'Counting on' for time calculations

Counting on in combination with a time line is an ideal method for calculating time intervals.

Set the scene with the following scenario:

- *The Vietnam War ended for Australian soldiers in 1975. How long ago was that?*

Scaffold the students with prompt questions, like those above, to draw the timeline. They may need a few more steps to help them cross the century and because the numbers are probably bigger than they are used to dealing with:

5 10 10 10 ?

1975 1980 1990 2000 2010 this year

The total time 5 + 10 + 10 + 10 + ? = 35 + ? years

Note: the last step depends on the year in which you are doing the activity.

Student may need less supporting steps, in which case it would look like:

5 20 10 ?

1975 1980 2000 2010 this year

A backwards example

Pose an example where the calculation would go from the current year and work back. For example:

- *My friend Susanna turned 43 in 2013; I want to know when she was born.*

Explain and fill in the diagram as you go:

- *This time we start at the current year (the other end) and draw the arrows back till we go 43 years back.*
- *For example, if it was 2013 the line would start there*

- *The first step would do the jump of 3 years back*
- *What year would that take us to?*
- *Then it is easier to go back 10 years at a time*
- *Stop when we have gone back 40 more years*

10 10 10 10 3

1970 1980 1990 2000 2010 2013

The year she was born was 1970

Together as a group go through the process of **checking** this subtraction by addition, using whichever method students are most comfortable with.

Ask: *Another friend, Mary, turned 27 at the same time. When was she born?*

Note: this time the jump from 2013 back to 2010 takes 3 years off the 7 but still leaves 4 single years to be stepped back at the end. See below.

4 10 10 3

1986 1990 2000 2010 2013

1986 + 27 = 2013

When were we born?

Ask students in turn to tell you how old they are now and get the other students to use the timeline counting backwards to calculate when they were born.

Further practice examples are provided *in Practice Sheet 3.*

How many hours?

Explain:

- *Often people who fill in time sheets at work have trouble calculating how long they worked*
- *For example, I worked from 10.00 AM to 1.30 PM.*
- *How many hours is that?*

Together build the following timeline:

1 1 1 ½

10 11 12 1 1.30

Hours I worked: $1 + 1 + 1 + \frac{1}{2} = 3\frac{1}{2}$ *hours*

Try a few more examples together:

- *Con worked from 9.30 AM to 2.30 PM. How many hours is that?*
- *Sammi worked from 10.45 AM to 4.15 PM. How many hours is that?*
- *Morrie worked from 9.30 AM for exactly 8 hours. What time did he finish?*
- *Jake worked from 10.15 to 2.30 how long was that?*

Follow up activity: How long ago was …?

Ask students to brainstorm some significant public events that they recall or have heard about in the past.

Write each of the events on a piece of paper and ask students to see if they can arrange the events in the order in which they happened. Stick them down on the paper as a reference.

Students should then take one event each and use the internet to find out its date. [You could do this yourself if it is not possible for the students.]

In the next class, use the dates collected to work out how many years ago each even occurred and so check whether the students' ordering was correct.

Counting on for subtraction

Practice Sheet 1

Use the counting on method to do these subtractions. Check your answers by adding.

1. 100 – 36
2. 200 – 36
3. 300 – 36
4. 700 – 36
5. 1,000 – 36

A group of friends is playing a game – they have to get 100 points to win. The table show how many points they have now. Use counting on to work out how many more each person needs.

	Person	Points now	Points to get 100
6.	Ani	47	
7.	Bev	26	
8.	Saed	53	
9.	Joni	61	
10.	Ahn	19	

Another game needs a total of 500 points to win. Use counting on to work out how many more each person needs.

	Person	Points now	Points to get 500
11.	Ani	120	
12.	Bev	248	
13.	Saed	304	
14.	Joni	115	
15.	Ahn	93	

How much further is it?

Practice Sheet 2

Use the Counting On method of subtraction to work out these distances.

Check all of your answers by adding.

Chen, Mani, Di and Susanna all have to drive their cars from Sydney to Melbourne along the Hume Highway.

The total distance is 881km.

1. Susanna stops for a last coffee break after 785 km. How many more kilometres does she have to drive?

2. Chen stops for the night after 450 kilometres. How far does he have to drive the next day to get to Melbourne?

3. Di has a lunch break when she has driven 562 km. How many kilometres does she still have to go?

4. Mani phones his mother after 608 km. How much further does he have to go?

5. The Hume Highway goes through Albury on the border of Victoria and NSW. It is 325 kilometres from Melbourne to Albury. How far is it from Sydney to Albury?

How long ago?

Practice Sheet 3

Use the time lines to help answer these questions. Check your answers by adding.

1. Australia changed to decimal currency in 1966. How many years ago was that?

1950	1960	1970	1980	1990	2000	2010	2020

2. Melbourne hosted the Olympics in 1956. How long ago was that?

1950	1960	1970	1980	1990	2000	2010	2020

3. Bella is 29 years old this year. What year was she born?

1950	1960	1970	1980	1990	2000	2010	2020

4. Moira is 64 years old this year. What year was she born?

1950	1960	1970	1980	1990	2000	2010	2020

5. Fred was born in 1965. How old is he now?

1950	1960	1970	1980	1990	2000	2010	2020

Addition with Useful Number Pairs

Overview

This activity uses the knowledge developed in the *Useful Number Pairs* Activity to demonstrate a quick technique that can be helpful when adding a collection of numbers.

The activity should be done after *Useful Number Pairs.*

Skills and Knowledge

Addition using pairs to 10

Addition using pairs to 100

Preparation and Materials

Photocopy Activity Sheet 1

Suggested Procedure

Start by revisiting the useful number pairs that add to 10.

Ask: *Write down three number pairs that add to 10.*

Collect students' answers till you have the whole list on the board.

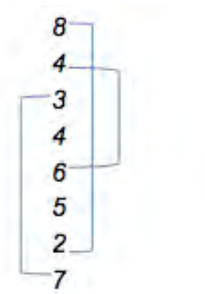
Explain:

- *These pairs can help you if you have a whole lot of numbers to add up*

Distribute Activity Sheet 1.

Ask:

- *Look at the first column of numbers*
- *Can you see any pairs that add to 10?*
  [Students should see 8+2, 4+6 and 3+7]
- *Draw lines between each of those pairs*
- *Now tell me what numbers are left*



The total is 10 + 10 + 10 + 4 + 5, which is 39.

Ask students to use the method of spotting the pairs to add the other collections of numbers on *Activity Sheet 1, examples 1 – 4.*

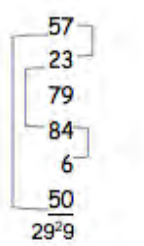
For each example ask:

- *How many pairs did you find?*
- *What numbers are left?*
- *What's the total?*

Using the pairs to 10 in columns

Explain that if you have a long addition with double digit figures, eg 59 + 63 + 21 etc. The pairs to 10 technique can be used, first in the units column and then again in the tens column.

For example:



Put the list of numbers on the board.

Ask:

- *Spot the pairs to 10 in the units column*
  [2 pairs of 10 + 9 = 29]
- *What will I write down?*
  [9 in units and 2 more in tens column]
- *Spot the pairs to 10 in the tens column*
  [2 pairs of 10 + 7 + 2 extras = 29 tens]
- *So the answer is?*
  [299]

See *Activity Sheet 1, examples 5, 6 & 7* for further practice using tens pairs in both columns.

Using pairs to 100

Put a quick example such as 99 + 40 + 60 on the board.

Ask: *Can anyone tell me quickly the answer to this?*

You are hoping someone will point out that 40 + 60 is a number pair that adds to 100.

So quickly they can see that only 99 is left to add and the quick answer is 199.

$$99 + \underbrace{40 + 60}$$

$$99 + 100 = 199$$

Ask:

- *Do you remember the pairs to 100?*
- *Can you give me some examples?*
- *What about the pairs that end in 5, e.g. 35?*

As students recall these, list them on the board for reference and to jog their memories.

Now ask students to try a few more examples quickly in their heads as you write them on the board. For example:

30 + 82 + 70 =
75 + 65 + 25 =
39 + 35 + 65 =
45 + 170 + 55 + 30 + 25 =

Note: In the last example the pair is actually 170 + 30 which adds up to 200. Students might need some extra practice on examples of this type.

When students seem confident ask them to try questions 9 – 18 on *Activity Sheet 1.* You may want to go over these one by one to ensure that students were able to spot the pair matching opportunities in each.

Pairs to 100 for adding money

The final examples on *Activity Sheet 1* are columns of prices similar to those that may be on supermarket dockets. Encourage students to use the pairs to 100 to add the cents and the pairs to 10 in the dollars.

Adding with useful number pairs

Activity Sheet 1

Add these columns of figures by first grouping the pairs of numbers that add to 10.

1.	**2.**	**3.**	**4.**
8	5	6	9
4	3	2	9
3	4	5	3
4	7	8	7
6	6	5	1
5	2	4	4
2	8	7	4
+ 7	+1	+9	+6

5.	**6.**	**7.**	**8.**
21	51	95	60
43	33	90	2
17	87	35	55
96	26	75	8
70	30	10	45
69	59	15	40
14	+74	+80	+70
+85			

9. 99 + 45 + 55

10. 75 + 59 + 25

11. 90 + 265 + 10

12. 35 + 65 + 310

13. 40 + 160 + 98

14. 25 + 85 + 15 + 75

15. 50 + 15 + 35 + 65 + 50 + 85

16. 70 + 140 + 230 + 60

17. 199 + 198 + 197 + 196 + 4 + 3 + 2 + 1

18. 75 + 65 + 55 + 45 + 135 + 225

19.	**20.**
85 c	$5.25
40 c	$3.50
35 c	$4.75
45 c	$7.05
65 c	$6.20
15 c	+$2.95
60 c	
+70 c	

Doubling Up

Overview

Doubling is a straightforward process that is useful for a range of in the head calculations involving multiplication by 2, 4 and 8. Because it relies on remembering only a few simple number facts, it can act provide a remarkable boost to students' numeracy confidence.

This activity describes strategies to assist students move from doubles of numbers from 0 to 10 to a method of doubling that can be applied to numbers and prices and prices of any size.

Skills and Knowledge

- Doubling single digit numbers
- Doubling tens and hundreds
- Doubling many digit numbers
- Multiplication by 4 & 8

Preparation and Materials

- Photocopy Practice Sheet 1 (1 per student)
- 10 sided dice
- Calculators (optional - 1 per pair of students)
- Make some practice sheets or questions cards from local advertising catalogues or newspapers: cut out individual items with their prices (some with dollar only prices and others with prices in dollars and small numbers of cents) and stick on paper or cards.

Suggested Procedure

Introducing doubling

Draw this table on the board

4	*8*
7	*14*
8	*16*
2	
6	
5	*10*
9	

Explain:

- *The same thing has being done to all of the numbers in column 1 to make the numbers in column 2*
- *Work out what it is and fill in the spaces*

After a few minutes ask:

- *What numbers did you put?*
- *What's happening to the numbers?*

Responses may include:

- *Add the number to itself*
- *Multiply by 2*
- *Times 2*
- *Double the number*

Establish that these processes would all give the same answer, so they are all the same thing really. For the purpose of this activity refer to it as 'doubling' the numbers.

Introduce the activity purpose

Ask:

- *Can you think of times when you would need to double numbers?*

Suggestions may include:

- Paying for two people, such as when buying tickets for a bus or cinema.
- Doubling the quantities in a recipe.
- Working out the total travelling time or distance if you are going to and from a place.

Explain:

- *We are going to look at strategies for doubling*
- *First small numbers and then larger ones*
- *Then you will see how you can use doubling for lots of other calculations*

Doubles of numbers from 0 - 10

Throw a 10 sided dice. Call the number thrown (e.g. 4)

Ask:

- *What is double 4?*

Record on the board:

4 + 4 = 8 and *double 4 = 8*

Repeat the procedure, encouraging quick recall from students and recording the new number doubles on the board as each is thrown.

As it arises, emphasise that double 0 is 0, since it is a common error to write 2.

Encourage students to memorise the doubles if they don't know them already.

Doubling larger numbers

Doubling any larger number can be done very simply by imagining that you are splitting the number into its simple components. These are all doubled separately then put back together as the final doubled number.

For example, 35 is split into 30 and 5. In diagram form this would look like:

35
↙ ↘
30 *5* *split up the number*
↓ ↓
60 *10* *double each number*
↘ ↙
70 *put the number back together*

Numbers such as 142 would be split into 100 and 40 and 2, each component doubled to 200, 80 and 4 then put together as 284.

For some students it may be necessary to backtrack and build up the steps of this process gradually by providing practice at doubling numbers such as 20, 30 , 40 .. followed by the hundreds: 200, 300, 400 .. (see below). However, it is good for students to see the point of learning the steps in advance by seeing the more complex examples first.

If any students are ready at this stage, the first column of Practice Sheet 1 provides examples of doubling numbers presented as prices.

Doubling tens and hundreds

Explain:

- *Once we can double these single numbers it's easy to double numbers with 0 on the end, like 20 or 30 … or even 200, 500 …*

Ask: *What do we get if we double 30?*

If 60 is not an automatic response ask:

- *What is 30 + 30?*

Remind students that adding 30 to itself gives the same answer as doubling.

Encourage students to work out a series of these calculations by adding until they see the pattern of doubling the tens number and then adding zero.

Double 20 is 20 + 20 = 40
Double 30 is 30 + 30 = 60
Double 40 is 40 + 40 = 80
Double 50 is 50 + 50 = 100 etc.

Once students are comfortable with doubling the tens, move on to the hundreds in a similar manner.

Sets of *Quick Questions* on cards are ideal for practising these with the whole class. For example

- *Double: 80; 30; 400; 60; 90*

See *Quick Questions* activity for further details about using Quick Questions.

If your students need reinforcement at different levels of complexity, create sets of 10 Questions for them to practise individually (see *10 Questions* activity).

Doubling and doubling again (x 4)

Ask:

- *What would I get if I double $7?*
- *What happens if I double it again?*
- *What else could I have done to $7 to get $28?*

Discuss and demonstrate:

$7 + $7 + $7 + $7 = $28

And also $28 = 4 x $7

Ask:

- *Do you think this would be the same if we doubled another number twice?*
- *Try it for $3*
- *What about $5?*

Let students experiment for themselves with small numbers until they are convinced that multiplying by 4 is the same as doubling twice. [They could use calculators if necessary.]

Explain:

- *You can use this method to find 4 times any number no matter how big it is*
- *You split them up the same way we did before*
- *This time you double the parts twice before you put them back together*
- *Try these examples by doubling twice:*

$32; $43; $125; $314; $509

Encourage students to check these answers, either by using calculators or adding 4 times.

Multiplying by 8

Ask: *Can anyone think of a way you could use this method to multiply by 8? (x 8)*

Use small numbers and the process described above to demonstrate that doubling 3 times is the same as multiplying by 8 or 'times' 8 (x 8). Get students to check the multiplication by 8 on calculators.

The second column of *Practice Sheet 1* (1- 5) provides practice at multiplying numbers by 4 using this method.

Doubling prices

Ask:

- *It costs $4.50 to get into the local swimming pool*
- *If I was paying for me and my sister how much would it cost?*

Demonstrate how this can be done by splitting the price into dollars and cents and doubling them separately

$4.50
↙ ↘
$4.00 50c *split up the price*
↓ ↓
$8 100c = $1 *double each part*
↘ ↙
$9.00 *put it back together*

Ask:

- *It costs $3.90 for a drink at our local café*
- *If I was paying for both me and my sister how much would it cost?*

Again work through the splitting process together.

This time the cents part comes to 180 cents so students may need help converting it to $1.80 and adding it to the $6.

The exercises in the first column of Practice Sheet 1 (11 – 15) ask students to apply this method to prices. In the second column they are asked for the cost of 4, which requires doubling a second time.

The cards prepared from the shopping catalogues can be used for further practice. For example, first ask students how much it would cost for two of each item, then follow up with costs for 4 or 8 of each.

Doubling Up

Practice Sheet 1

Use doubling in your head or diagrams to work out these costs:

	Cost for 1	Cost for 2	Cost for 4
1.	$5		
2.	$9		
3.	$11		
4.	$15		
5.	$67		

A business wants to buy two of each of these appliances. Double each of the prices

6. Electric Fan
$98

7. Coffee Machine $234

8. LCDTV only $517!!

9. Air conditioning for your home or office.

Just $2,900

10. Family sized refrigerator now only **$709**

Use doubling in your head or diagrams to work out these costs:

	Cost for 1	Cost for 2	Cost for 4
11.	50c		
12.	75c		
13.	$3.20		
14.	$4.45		
15.	$7.50		

The Power of Halving

Overview

The skill of halving numbers is very useful for a range of in the head calculations, including division by 2, 4, and 8.

Since many students find division one of the most difficult skills to master, this can be extremely helpful to them. Like doubling, halving relies on only a few number facts, so these strategies can make a powerful contribution to building students' confidence at in the head calculations.
Halving is also particularly useful for finding ½, ¼, and ¾ and calculating 50%, 25% and 75%

This activity should be done after *Doubling Up.* It links directly to the activities *Sorting Fractions of ..* and *Shortcut Percentages 50%, 25% & 75%.*

Skills and Knowledge

- Halving even numbers
- Halving odd numbers
- Division by 2, 4 & 8
- Halving prices

Preparation and Materials

- Write the even numbers as dollars from 0 – $20 on pieces of paper the size of flashcards.
- Make a similar set of prices using odd numbers from $1 - $19.
- Photocopy Practice Sheet 1(1 per student)
- Prepare sets of 10 Questions as needed

Suggested Procedure

Revisiting the doubles of single digit numbers

Write all the numbers from 0 – 10 in a column on the board

Ask: *What are the doubles of all these numbers?*

Record the doubles on the board beside the numbers.

Number	Double
0	0
1	2
2	4
3	6
.	.
.	.
.	.

Halving numbers from 0 – 20

Ask:

- *Imagine you are sharing a snack with a friend and you want to pay half each.*
- *It costs $8*
- *How much is a half of $8?*

Record this on the board beside 'double 4 = 8':

Double 4 = 8 *½ of 8 = 4*

Emphasise the relationship between these two opposite (inverse) operations:

- *Halving is just going backwards from the double numbers*

Diagrams can also be helpful to emphasise this relationship.

Either: $$4 \underset{\text{halve}}{\overset{\text{double}}{\rightleftarrows}} 8$$ $$4 \underset{\div\, 2}{\overset{\times\, 2}{\rightleftarrows}} 8$$ or:

Using
flashcards prepared with the even numbers from 0 to $20 hold them up one at a time in random order.

Ask:

- *Write down half of these prices as I hold them up*

Keep going until you are confident that students have a good recall of the halves of the even numbers.

The flashcards could be used at other times as revision.

Because doubling is the opposite of halving it should always be used as a checking strategy. The sooner students are introduced to this way of thinking the better.

Halving odd numbers

Ask: *If you share $3 between two people how much would each person get?*

If students are not sure of this then act out the process with three dollar coins.

- *Each person gets one whole dollar*
- *What about the dollar left over?*
- *Yes you change it into two 50 cent pieces*
- *How much does each person get?*

Together try a few more examples, such as sharing $5, $7, $11, then use the second set of flashcards until students are confident about the extra 50 cents in all of these examples.

Halving tens and hundreds

The halving process of the even numbers such as 40, 60, 80 … and 200, 400, 600 … can be introduced as the reverse process of doubling tens and hundreds and should require only a little practice. Use sets of numbers on flashcards or the board, making sure the number in the front (leading digit) is even.

Numbers such as 30, 50, 70 and 300, 500, 700, with odd leading digits may need to be looked at separately. SEE BELOW.

Again, imagine the sharing process, beginning with:

- *Two people get half of $10 – how much do they each get?*
- *Two people get half of $30 – how much do they each get?*

Explain:

- *We can split the $30 into $20 and $10 – these are both easy to halve*

Draw the diagram of the process

$30

↙ ↘

$20 $10 *split the number*

↓ ↓

$10 $5 *halve each part*

↘ ↙

$15 *put back together*

Try several more examples together until students realise that they all end in 5 and can do the process quite quickly.

Slip in a few examples of hundreds, such as, *$500, $300, $700* as you do these examples.

Sets of *10 Questions* on paper, or flashcards of *Quick Questions* at the beginning of following sessions will help students to master and recall these processes.

Once students are confident with the single digit numbers described above, they should be able to combine them to halve many digit numbers.

For example, try these together:

Find a half of: $25, $63, $92, $230; $365

Practice Sheet 1: Halving money provides some student practice.

Halving and halving again (÷ 4)

Just as the doubling process can be used twice to multiply by 4, halving twice is the equivalent of dividing by 4.

Since many students find division one of the most difficult skills to master, this may be greeted quite enthusiastically.

To introduce this, start with a number easily halved twice in the head, such as 8.

Ask:

- *What is half of 8?*
- *What happens if halve it again?*
- *What's half of 4?*

Demonstrate that you would get the same result by dividing by 4 since *8 ÷ 4 = 2.*

Students could check this with a calculator or compare it with its opposite 4 x 2 = 8.

Encourage students to experiment with a few more numbers which are straightforward to halve twice.

Ask:

- *Use the method of splitting and halving the parts to find half of:*

  36; 48; 100; 288; 1,012

Model the strategies for students and compare any variations suggested by them.

> *Encourage students to check by using the opposite process of multiplying by 4 or doubling twice.*
> *For example, if halving 288 twice gives the answer 72, they should check by going backwards with 72 X 4 = 288 or double 72 = 144 and double 144 = 288.*

Try some slightly more complex examples which require re combining and/or re-splitting during the two halving steps, such as:

52; 132; 304

Further Practice is available as sets of 10 questions in *Practice Sheet 2.*

Practice Sheets 1 and 2 of the activity *Sorting Fractions of…* provide further practice examples of finding ½ and ¼ of assorted prices.

Half Price Sale!!

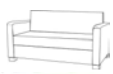
Calculate the costs of these items.

1.  *$7* New price:

2.  *$18* New price:

3.  *$60* New price:

4.  *$400* New price:

5.  *$70* New price:

6.  *$900* New price:

Anya goes out for the day with her flatmate. They pay half each of their costs.
What does she pay for these?

			Anya's share
7.	The taxi	$16	
8.	Snacks	$23	
9.	Drinks	$31	
10.	A birthday present for a friend	$52	
11.	A new dining table for their flat	$270	

Jake and a friend run a part-time mowing business.
They split the takings half-half.
How much did Jake make each day?

		Total takings	Jake's half
12.	Monday	$250	
13.	Tuesday	$165	
14.	Wednesday	$233	
15.	Thursday	$507	
11.	Friday	$391	

Halving twice

Practice Sheet 2

✂ Cut these and give to students as single sets of examples.

Set 1

Use the method of half then half again to do these divisions.

1. 32 ÷ 4	6. 848 ÷ 4
2. 64 ÷ 4	7. 460 ÷ 4
3. 120 ÷ 4	8. 500 ÷ 4
4. 320 ÷ 4	9. 684 ÷ 4
5. 820 ÷ 4	10. 160 ÷ 4

Check your answers by doubling twice.

Set 2

Use the method of half then half again to do these divisions.

1. 36 ÷ 4	6. 484 ÷ 4
2. 100 ÷ 4	7. 640 ÷ 4
3. 360 ÷ 4	8. 300 ÷ 4
4. 204 ÷ 4	9. 416 ÷ 4
5. 240 ÷ 4	10. 108 ÷ 4

Check your answers by doubling twice.

Set 3

Use the method of half then half again to do these divisions.

1. 108 ÷ 4	6. 600 ÷ 4
2. 52 ÷ 4	7. 804 ÷ 4
3. 280 ÷ 4	8. 700 ÷ 4
4. 128 ÷ 4	9. 150 ÷ 4
5. 816 ÷ 4	10. 250 ÷ 4

Check your answers by doubling twice.

Splitting Numbers for Addition & Multiplication

Overview

This activity explores an alternative, common sense method for in the head addition which relies on the technique of 'splitting numbers' into tens and units, used in *Doubling Up*, and *The Power of Halving*. This method can also be extended to make sense of multiplication of larger numbers.

Skills and Knowledge

Addition using in the head techniques

Preparation and Materials

Photocopy Practice Sheet 1

Suggested Procedure

Introducing the activity

Put the question in context with a scenario.
For example:

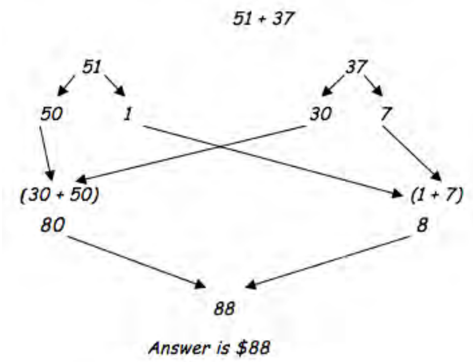
- *My friend Stephanie does odd jobs for people. She always does her calculations in her head. On Monday she earned $51 and on Tuesday she earned $37.*
- *How might she have calculated it without writing it down like she was taught in school?*

Follow up on any alternate strategies suggested by students before demonstrating the splitting up method, as follows:

In her head she splits the numbers into tens and units:

Then she combines the tens: 50 + 30 and the ones: 1 + 7.
Then she puts the split number back together again.

The process can be clarified on the board with a rough diagram. For example:

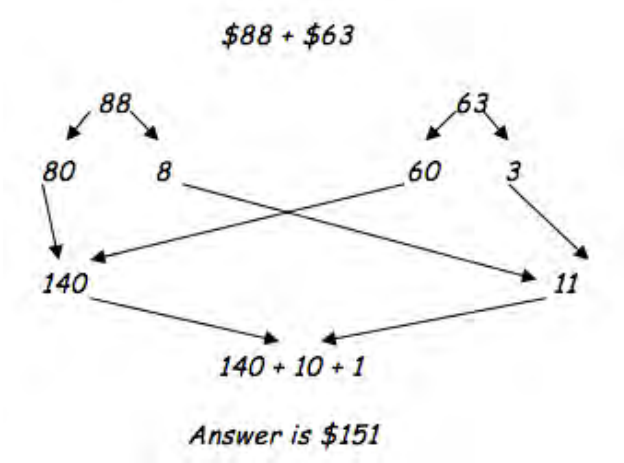


Explain to students that it is not necessary to draw all of the arrows. In fact, they can probably imagine the whole process in their heads, but sometimes it is safer to jot down some of the intermediate steps on a scrap of paper.

Try another example together:

On Wednesday Stephanie earned another $63. Work out how much she has earned so far?

Encourage students to use the process, first by themselves, then compare on the board. Clarify the process again using a diagram.

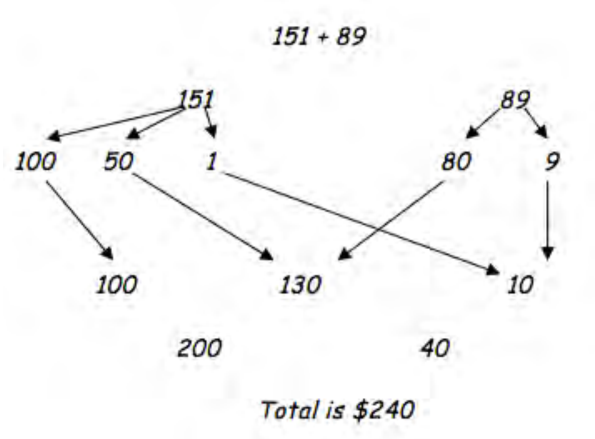


Extending into hundreds

Try another example to demonstrate that sometimes the numbers can also be split into three parts. Use the odd job earnings again to demonstrate.

Ask: *For the rest of the week Stephanie earned $89. What's her total now?*

Break the 151 into three parts and proceed as before. Some students may need more steps written than others:



Give students plenty of time in class to practice this technique. Some graded examples (in groups of 4) are provided as models in *Practice Sheet 1.*

To boost their confidence make sure students get plenty of practice at each of the levels, by creating further examples yourself, rather than letting them try more difficult examples too soon.

Splitting the number for multiplication

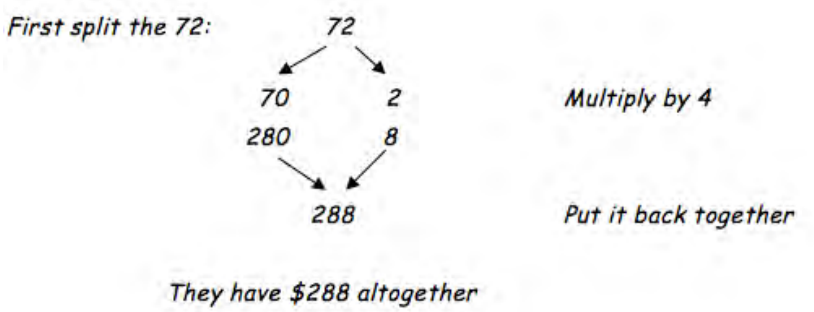
This splitting the number technique is a way that students might make more sense of multiplication that using the traditional school rote method.

One example can be used to demonstrate, followed by opportunities for further practice together.

Example:

- *4 people pay $72 each to help pay for a birthday party for their mother.*
- *How much to they have altogether?*

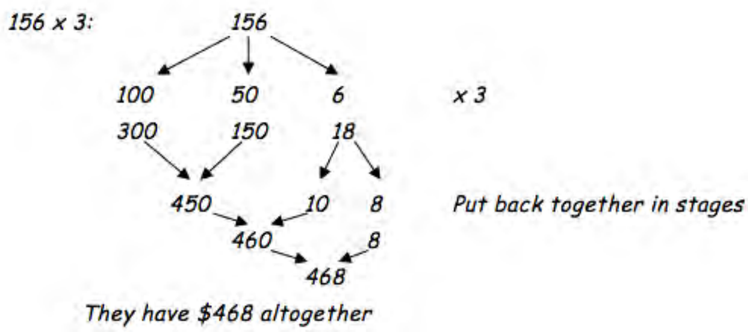
On the board: *$72 x 4 = ?*



Together try another example, this one with hundreds in it, to be split into three parts. For example:

3 people win $156 each how much do they have altogether?

The number of steps involved will depend on how easily students can 'see' the split numbers. The diagram below represents one version only.



Practice several more examples together, encouraging students to jot down only the in between numbers that they need to.

Then discuss with them whether this method is any clearer or quicker for them than standard multiplication methods.

Further examples are provided in *Practice Sheet 1.*

Splitting numbers

Practice Sheet 1

Do these additions by splitting the numbers

1. 43 + 56

2. 35 + 24

3. 25 +73

4. 17 + 22 + 30

5. 62 + 18

6. 74 + 26

7. 87 + 62

8. 58 + 32

9. 175 + 214

10. 329 + 143

Illustration by Elise Gueyne

Splitting numbers

Do these additions by splitting the numbers

11. 436 + 183

12. 209 + 152 + 30

Try these harder additions by splitting the numbers

13. 384 + 981

14. 411 + 659

15. 413 + 794

16. 805 + 1,106

Try these multiplications by splitting the numbers

17. 54 x 3

18. 217 x 4

19. 207 x 5

20. 126 x 6

Multiplying by Tens

Overview

The shortcuts of adding or removing zeros when multiplying and dividing by 10, 100 and 1,000 are extremely important in a society that uses decimal (10-based) systems of currency and measurement. They are also very useful for approximation of calculations, whether for making predictions or estimates, or for checking results given by spread sheets and calculators. These are skills that are often taken for granted in our society but which may need specific attention for adult numeracy students.

This activity introduces these important skills for multiplication in gradual steps using an approach that allows adult students to recognise the patterns then apply the shortcuts to a range of whole number calculations.

It also extends the skills to multiplication by numbers such as 20, 30, 50 … and 200, 300, 500 ...

Skills and Knowledge

Shortcut multiplication by:

- 10
- 100
- 20, 30 …
- 200, 300 …

Preparation and Materials

Photocopy Practice Sheets 1 & 2
(1 per student)

Set of basic calculators

Suggested Procedure

Introducing the x 10 pattern

To set these multiplications in some context, ask:

- *Drinks are $4 a glass at a fundraising BBQ*
- *How much will 10 glasses cost?*

Allow students time to work this out using their own strategies. Then discuss different methods by they arrived at their answers.

These could include:

- 4 + 4 + 4 …
- Use of a written multiplication table
- Use of a calculator
- Adding a zero

Record on the board: 10 x $4 = $40

Similarly for the next few questions – discuss the method then record the answers.

- *Hamburgers sell at $6. How much will 10 cost?*
- *A vegetable curry is $9. How much for 10?*

You will have now recorded: *10 x 4 = $40*
10 x 6 = $60
10 x 9 = $90

Ask:

- *Do you see a pattern here?*
- *Without working out can you predict the answers to:*
  - *10 x 3?*
  - *10 x 5?*
  - *10 x 8?*

Extending the shortcut to larger numbers

Discuss with students whether they think this shortcut will work for bigger numbers, with more digits, for example 45, or numbers with more zeros, such as 400.

Ask:

- *Use the shortcut to do these calculations*
- *Test your answer using a calculator:*
  - *20 x 10*
  - *35 x 10*
  - *300 x 10*
  - *704 x 10*
  - *6,080 X 10*

Keep going until students are confident that the shortcut works for any whole numbers.

Practice Sheet 1 Contains sets of short questions to help students gain confidence with these skills. Set 1 focuses on multiplication by 10.

Extending to x 100

Once multiplying by 10 is established move on to discuss the following sequence of questions. Encourage students to use calculators in addition to any ot her methods they might choose for these calculations.

Ask:
What if 100 of everything is sold?

- *100 drinks at $4*
- *100 hamburgers at $6?*
- *100 vegetable curries at $9?*

Record: *100 x 4 = 400*

100 x 6 = 600

100 x 9 = 900

Again ask:

- *Can you see a pattern?*
- *Can you predict the answers to these:*
    - *100 x 3 = ?*
    - *100 x 7 = ?*
    - *100 x 8 = ?*

Continue until the rule of adding two zeros is established.

Extending the shortcut to other numbers

As with multiplication by 10, get students to test the shortcut with a variety of whole numbers and to check what they get with a calculator to be sure for themselves that the shortcut does work.

Examples could include:

- *100 x 30*
- *52 x 100*
- *100 x 360*
- *4,000 x 100*
- *6,721 x 100*

Practice Sheet 1 Contains sets of short questions to help students gain confidence with these skills. Set 2 focuses on multiplication by 100.

Extending to x 20, 30 … or 200, 300 …

These calculations are straightforward once students are aware that the multiplication can be done in two simple steps.

For example, since:

- 20 = 2 x 10, multiplying by 20 is the same as first multiplying by 2, then by 10 or vice versa.
- 300 = 3 x 100 multiplying by 300 is the same as first multiplying by 100, then by 3.

Again it is a matter of letting students experiment by trying out the possible shortcut then checking that it works with a calculator.

You may wish to set it in a context to begin the discussion and show the steps in way which will possibly be instinctive to students.

Ask:

- *Drinks at a concert cost $4 each*
- *I sold 20 drinks in the first few minutes*
- *How much money should I have?*

Encourage students to try their own methods then discuss the possibility of looking at 10 drinks at a time:

- *1 drink costs $4*
- *10 drinks costs $4 x 10 = $40*
- *20 drinks is double 10 or (2 x 10)*
- *So double $40 = $80 or (2x$40 = $80)*

Get students to check $4 x 20 using calculators to see that it gives the same result as the two stage process.

Try a few more similar questions with say, 30 drinks, then similar multiples of other items with single digit prices. Include quantities such as 200, 400 in the investigations.

Practice Sheet 1 Contains sets of short questions to help students gain confidence with these skills. Set 3 focuses on multiplication by multiples of 10s and 100s.

Practice Sheet 2 Contains questions that involve using all of these shortcut multiplication skills applied to realistic situations.

Further Practice
Sets of *Quick Questions* at the beginning of following sessions would be a good way to keep reinforcing these skills.

Possible extension
The same technique, of recognising a pattern emerging after several calculations with a calculator, can be used to extend these shortcut techniques to situations which involve decimals, particularly prices with dollars and cents.

x 10 and 100

Practice Sheet 1

Set 1

Write answers to these using shortcuts only. When you have finished check answers with a calculator.

1. 9 x 10	6. 10 x 90	11. 10 x 91
2. 15 x 10	7. 970 x 10	12. 702 x 10
3. 10 x 27	8. 30 x 10	13. 10 x 35
4. 36 x 10	9. 101 x 10	14. 10 x 305
5. 45 x 10	10. 10 x 200	15. 17 x 10

Set 2

Write answers to these using shortcuts only. When you have finished check answers with a calculator.

1. 2 x 100	6. 450 x 100
2. 5 x 100	7. 100 x 720
3. 100 x 7	8. 3,000 x 100
4. 62 x 100	9. 90 x 100
5. 101 x 100	10. 100 x 5,020

Set 3

Write answers to these using shortcuts only. When you have finished check answers with a calculator.

1. 50 x 30	6. 100 x 40	11. 600 x 90
2. 40 x 20	7. 300 x 40	12. 1,200 x 20
3. 30 x 90	8. 200 x 20	13. 110 x 40
4. 50 x 40	9. 70 x 50	14. 30 x 500
5. 60 x 50	10. 400 x 60	15. 700 x 30

x 10 and 100

Practice Sheet 2

Write answers to these problems.

1. Sam's trip from home to work is 4km. How far would he travel if he did the trip:

  - 10 times?
  - 20 times?

2. There are 8 muesli bars in a packet. Sonya bought 20 packets for a children's picnic?

  - How many muesli bars did she get?

3. It takes Rosa 2 minutes to address one envelope. How many minutes will she take to address:

  - 10 envelopes?
  - 30 envelopes?
  - 100 envelopes?

4. There are 30 Vitamin C tablets in a bottle.
   If you buy 4 bottles, how many tablets will there be?

5. Maria types 60 words a minute. How many words will she have typed after typing for:

  - 100 minutes?
  - One hour?

6. A ream (packet) of paper contains 500 sheets. How many sheets in:

  - 6 reams?
  - 10 reams?

7. Janine earns $20 an hour as a waitress. How much would she earn after:

  - 10 hours?
  - 20 hours?

Division by Tens

Overview

The shortcut of removing zeros when dividing by 10, 100 and 1,000 is extremely important in a society that uses decimal (10-based) systems of currency and measurement. It is also very useful for approximation of calculations, and for in the head methods of calculating 10%. This is a skill which may need specific attention for adult numeracy students.

This activity introduces these important skills for division in gradual steps using an approach that allows adult students to recognise the patterns then apply the shortcuts to a range of whole number calculations.

It also extends the skills to division by numbers such as 20 and 200 by combining halving with the division by tens, and to 30, 40, 300, 400 ... for students comfortable with other short divisions.

Skills and Knowledge

Shortcut division by

- 10 & 100
- 20 & 200
- 30, 40 & 300, 400 (optional)

Preparation and Materials

Photocopy Practice Sheet 1 (1 per student) cut into sets of 10 questions.

Practice Sheet 2 as required

Set of basic calculators

Suggested Procedure

Division by 10

Begin by setting the skill in a meaningful context.

Ask:

- *I pay $20 for a packet of 10 pencils*
- *How much am I paying for each pencil?*

Allow time for students to consider their own method for working this out before discussing it with the group.

Strategies may include:

- Guessing and checking by repeated addition
- Guessing and checking by multiplication
- Repeated subtraction
- Use of a calculator

All of these alternative strategies are valid.

Adults whose numeracy is not strong often avoid division, instead using their own idiosyncratic but creative methods. These should be respected and shared at the same time as the new much quicker skills are developed.

It is important for the purpose of this activity that students realise that this is a division calculation and is represented by the ÷ symbol. So write prominently on the board:

$20 ÷ 10

This could also be highlighted by asking all students to check their answer using the division button on the calculator.

Record the answer on the board before repeating the procedure with a couple of other questions.

$20 ÷ 10 = $2

For example, ask:

- *10 people who work together share a lottery ticket*
- *It wins $80.*
- *How much should they each get?*

With 3 or 4 examples on the board, for instance:

$20 ÷ 10 = $2

$80 ÷ 10 = $8

$60 ÷ 10 = $6

Ask:

- *Can you see a pattern?*
- *Can you answer these using the pattern?*

    - *$70 ÷ 10 =*
    - *$50 ÷ 10 =*
    - *$30 ÷ 10 =*

Multiplication and division as opposites

This would be a good time to discuss the relationship between multiplication and division: that they are the opposites (inverses) of each other.

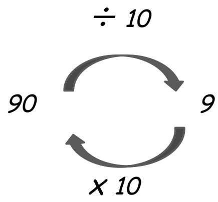
To highlight this ask:

- *Pick any number you like ending in 0*
- *Write it down*
- *Divide it by 10 using the shortcut*
- *Now multiply it by 10 using the shortcut*
- *What happened?*

Ask:

- *Do the calculations again using the calculator*
- *Is the result the same?*
- *Try again using another number*

The inverse nature of the operations is important because multiplication is the best means of checking division calculations. The relationship can be illustrated using a diagram:



Extending the shortcut to larger numbers

Discuss with students whether they think this shortcut will work for bigger numbers, for example, numbers with more zeros such as 200, or 3,000.

Ask:

- *Use the shortcut to do these calculations*
- *Test your answer using a calculator:*

    - *200 ÷ 10*
    - *400 ÷ 10*
    - *3,000 ÷ 10*
    - *7,000 ÷ 10*
    - *80,000 ÷ 10*

Keep going until students are confident that the rule applies for all of these whole numbers.

Practice Sheet 1 contains sets of short questions for students to practise this shortcut. Set 1 focuses on division by 10. These are best cut into sets and used at different times as revision, rather than all done at one time.

Five quick questions on half A4 paper, at the beginning of follow up sessions is also a very effective way to reinforce this skill.

Division by 100

This could be approached as an extension of the pattern of dividing by 10 with students predicting the answers and checking on calculators.

Alternatively, it could be put into a context that allows students to think through the process first. This also reinforces the concept of division and situations in which it is used.

Pose the question:

- *Roger receives a payment of $400 at the bank*
- *He asks to be paid in $100 notes*
- *How many notes will he get?*

Take a few minutes for students to reflect on their answers and individual methods. Discuss these then go on to frame this as a division calculation if this has not already come from students.

Ask:

- *How would we write this as a calculation?*
- *What symbols would I have to use?*

On the board record:

$400 how many $100

→ $400 ÷ $100 = 4

Get students to check that a calculator would give the same result.

To build the pattern, vary the question with different amounts of money.

For example:

- *$500 ÷ $100*
- *$700 ÷ $100*
- *$300 ÷ $100*

Again, record these with answers on the board until the pattern of removing two zeros is established.

Practice Sheet 2, Sets 4 & 5 provides practice questions for this skill. Further examples can be generated as Quick Questions on flashcards to revise at the beginning of future sessions.

Dividing by 20

Division by 20 can be seen as a two-step process, first halve then divide by 10 or vice versa.

To make sense of this an example in context may help.

Pose the question:

- *A $600 food or drink bill for a party is shared between 20 people*
- *How much does each person pay?*

Explain:

The calculation can be split into something easy to do in the head by first splitting the group in half: 10 people in each group and $300 to divide between them

$300 shared by 10 people → $300 ÷ 10 = $30

$600

$300 shared by 10 people → $300 ÷ 10 = $30

Each person pays $30

So $600 ÷ 20 = $30

To consolidate the idea, ask students to use this 2 step method to calculate:

- *How much would each of the 20 people pay if the bill was:*
  - *$800*
  - *$600*
  - *$700*
  - *$500*

Changing the order of the steps

In some calculations it is more convenient to divide by 10 prior to halving if students are not yet comfortable with decimals.

For example 150 ÷ 20:

Halving first → 75 which is then difficult to divide by 10 if students are not yet comfortable with decimals.

Dividing by 10 first → 15 which can be split into 10 and 5 for halving
→ ½ of 10 = 5 and ½ of 5 = 2 ½
Result is 5 + 2½ = 7½

To get students comfortable with the idea that the steps can be done either way, explore the idea with them by experimenting.

Ask:

- *When we calculated $600 ÷ 20 we halved the $600 first*
- *Do you think you would get the same result if you divide by 10 before you halve?*
- *Can you try it to find out?*

Encourage students to try all the examples they have done so far by reversing the procedure. Explain that sometimes it is easier to do one way first, sometimes the other.

Dividing by 200

Division by 200 can be done using the same two-step process, using examples such as:

- *How many payments of $200 could be made from $400; $800; $1,200.*

These of course translate as:

- *$400 ÷ $200*
- *$800 ÷ $200*
- *$1200 ÷ $200*

Create a set of 10 practice questions for students if they want to practice this skill further.

Division by 30, 40 … 300, 400 …

These skills are, on the whole, most useful for estimation and approximations of calculations involving large numbers. So if students will be progressing to higher levels of numeracy it is worth extending the activity to include them.

Although division by 20 could be done by the simple process of halving and then dividing by 10, division by other tens numbers relies on students being able to divide by single digit numbers, a skill less easily developed than halving.

The process is similar to that outlined above. To avoid tricky situations with decimals and/or remainders it is better that the division by 100 step is done first.

Practice Sheet 2 provides some examples in which the division is easily recognisable. These should give students confidence with the method, even if they have yet to develop their divisions skills.

÷ by 10

Practice Sheet 1

✂ Cut these and give to students as single sets of examples.

Set 1

Write answers to these using shortcuts only. When you have finished check answers with a calculator.

1. 70 ÷ 10
2. 700 ÷ 10
3. 7,000 ÷ 10
4. 8,000 ÷ 10
5. 900 ÷ 10
6. 30 ÷ 10
7. 300 ÷ 10
8. 3,000 ÷ 10
9. 320 ÷ 10
10. 3,200 ÷ 10

Set 2

Write answers to these using shortcuts only. When you have finished check answers with a calculator.

1. 80 ÷ 10
2. 900 ÷ 10
3. 60 ÷ 10
4. 4,000 ÷ 10
5. 1,200 ÷ 10
6. 3,040 ÷ 10
7. 720 ÷ 10
8. 1,020 ÷ 10
9. 15,000 ÷ 10
10. 1,560 ÷ 10

Set 3

Write answers to these using shortcuts only. When you have finished check answers with a calculator.

1. 40 ÷ 10
2. 4,000 ÷ 10
3. 600 ÷ 10
4. 50 ÷ 10
5. 1,20 ÷ 10
6. 1,030 ÷ 10
7. 6,100 ÷ 10
8. 620 ÷ 10
9. 1,400 ÷ 10
10. 3,250 ÷ 10

÷ by Multiples of 10

Practice Sheet 2

✂ Cut these and give to students as single sets of examples.

Set 4

Write answers to these using shortcuts only. When you have finished check answers with a calculator.

1. \$900 ÷ \$10
2. \$900 ÷ \$100
3. \$300 ÷ \$100
4. \$2,000 ÷ \$100
5. \$14,000 ÷ 10
6. \$1,200 ÷ \$10
7. 12,000 ÷ 100
8. 4,300 ÷ \$100
9. 500 ÷ 100
10. 10,200 ÷ 100

Set 5

Write answers to these using shortcuts only. When you have finished check answers with a calculator.

1. \$500 ÷ \$100
2. \$3,500 ÷ \$1000
3. 700 ÷ 100
4. 6,000 ÷ 100
5. \$2,900 ÷ \$100
6. 102,000 ÷ 100
7. \$40,000 ÷ 100
8. 50,100 ÷ 100
9. \$2,000,000 ÷ \$100
10. \$10,100 ÷ 100

Set 6

Write answers to these using shortcuts only. When you have finished check answers with a calculator.

1. \$600 ÷ \$10
2. \$600 ÷ \$20
3. \$300 ÷ \$100
4. \$1,600 ÷ \$20
5. \$4.000 ÷ 20
6. \$1,800 ÷ \$100
7. 12,000 ÷ 20
8. 4,300 ÷ \$20
9. 500 ÷ 100
10. 10,200 ÷ 200

÷ by Multiples of 10

Practice Sheet 2 (cont.)

✂ Cut these and give to students as single sets of examples.

Set 7

Write answers to these using shortcuts only. When you have finished check answers with a calculator.

1. 900 ÷ 10
2. 900 ÷ 30
3. 1,200 ÷ 10
4. 1,200 ÷ 40
5. 1,200 ÷ 60
6. 1,500 ÷ 30
7. 1,500 ÷ 50
8. 2,400 ÷ 30
9. 2,400 ÷ 40
10. 6,000 ÷ 30

Set 8

Write answers to these using shortcuts only. When you have finished check answers with a calculator.

1. 800 ÷ 400
2. 2,700 ÷ 900
3. 2,000 ÷ 500
4. 2,500 ÷ 500
5. 2,100 ÷ 300
6. 2,100 ÷ 700
7. 3,200 ÷ 400
8. 3,200 ÷ 800
9. 14,000 ÷ 700
10. 16,000 ÷ 800